A fast approach to estimating Windkessel model parameters for patient-specific multi-scale CFD simulations of aortic flow

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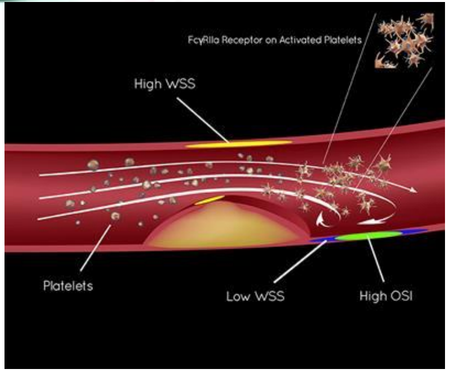
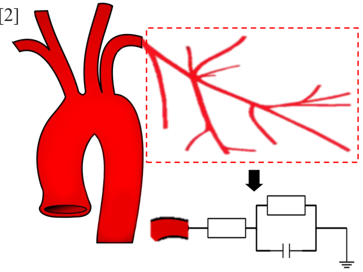
Introduction:

- Knowing hemodynamics in vessel is important.
  - Wall shear stress (WSS) is related to thrombosis.
- Current clinical flow visualization techniques have limitations.
  - Difficult to obtain velocity and pressure fields in vasculature.
- Computational fluid dynamics (CFD) with Windkessel model.
  - Current way to find parameters for Windkessel model needs iterations of periodic flow simulation.
- Objective: find a fast way to obtain accurate Windkessel model parameters.



[1]

[2]

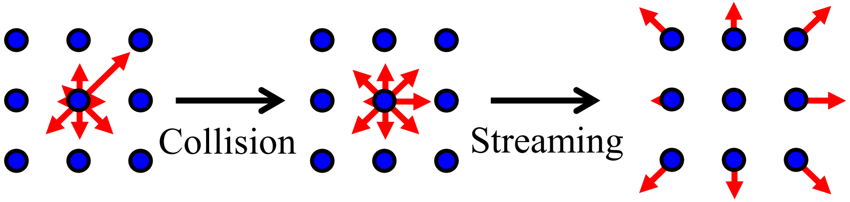


Lamped parameter model

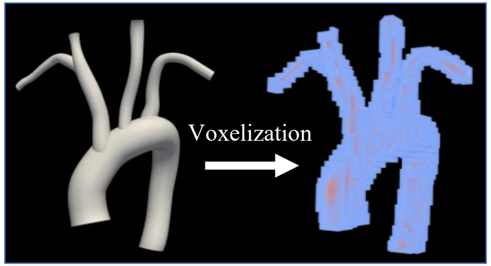
[1] Liebeskind D S, Hinman J D, Kaneko N, et al. Endothelial Shear Stress and Platelet FcγRIIa Expression in Intracranial Atherosclerotic Disease[J]. Frontiers in Neurology, 2021, 12: 244.
[2] https://en.wikipedia.org/wiki/Ascending_aorta

Lattice Boltzmann method (LBM)

- Simple two step algorithm:
  1. Fluid node collides by the collision operator.
  2. Stream to neighbor fluid nodes.

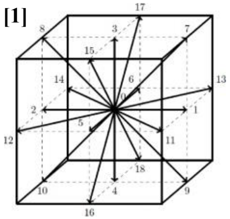


- Advantages:
  - Collision step is local; streaming step takes no computation; natural to parallelize.
  - Easy to handle complex geometry by voxelization.



- BGK collision operator and D3Q19 lattice structure are used in this study.

[1]



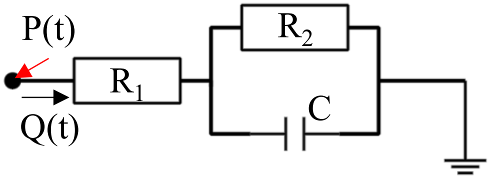
D3Q19 descriptor

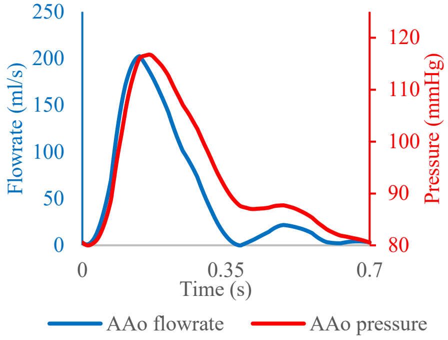
[2]

BGK operator

[1] Lenan, Zhang & Jebakumar, Anand Samuel & Abraham, John. (2016). Lattice Boltzmann method simulations of Stokes number effects on particle motion in a channel flow. Physics of Fluids. 28. 063306. 10.1063/1.4953800.
[2] https://en.wikipedia.org/wiki/Bhatnagar%E2%80%93Gross%E2%80%93Krook_operator

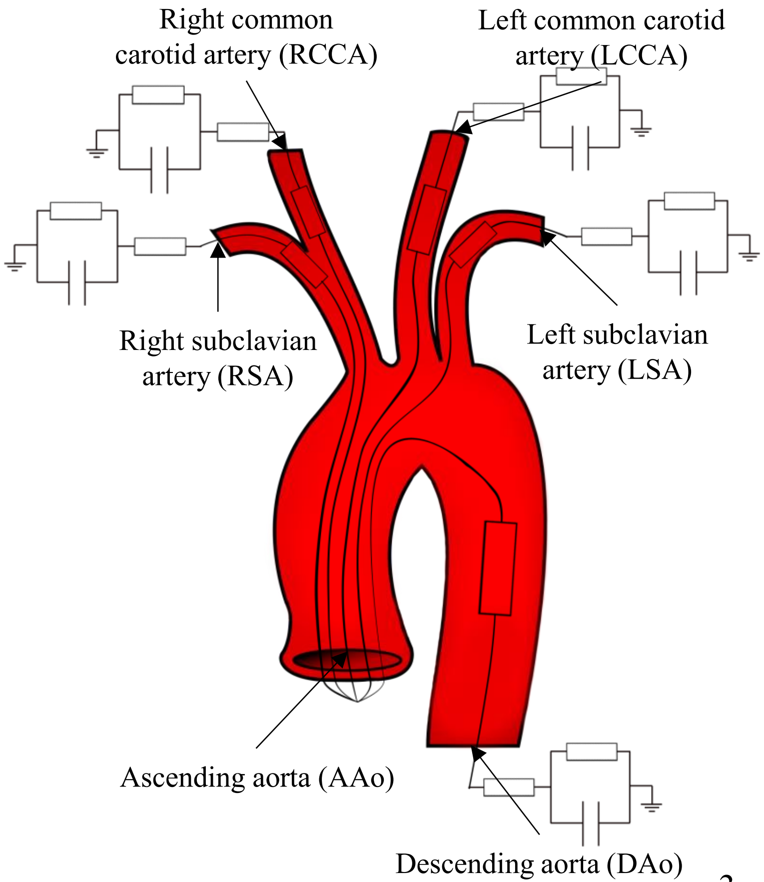
Three element Windkessel model





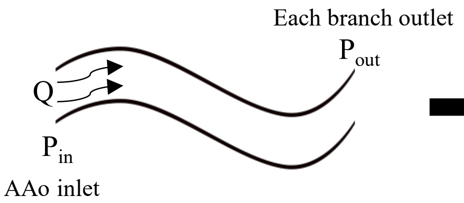
$$\left(1+\frac{R_1}{R_2}\right)Q(t)+CR_1\frac{\partial Q(t)}{\partial t}=\frac{P(t)}{R_2}+C\frac{\partial P(t)}{\partial t}$$

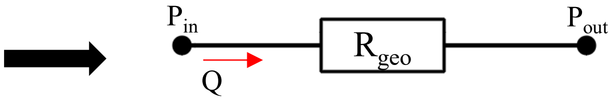
R_1: characteristic resistance; **R_2**: peripheral resistance; **C**: compliance;



Aorta figure from: https://en.wikipedia.org/wiki/Ascending_aorta

Geometric resistance





Electrical circuit analog

$$R_{geo}=\frac{\int_0^T [P_{in}(t)-P_{out_i}(t)]dt}{\int_0^T Q_{out_i}(t)dt}$$

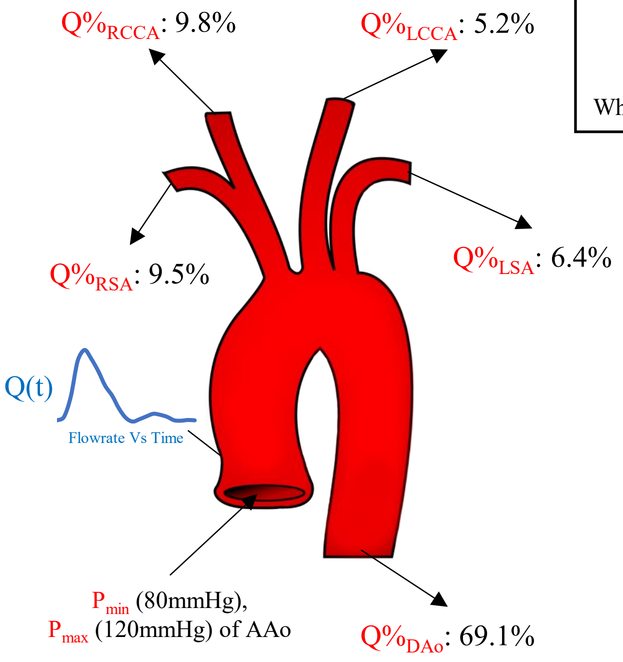
Goal of our algorithm

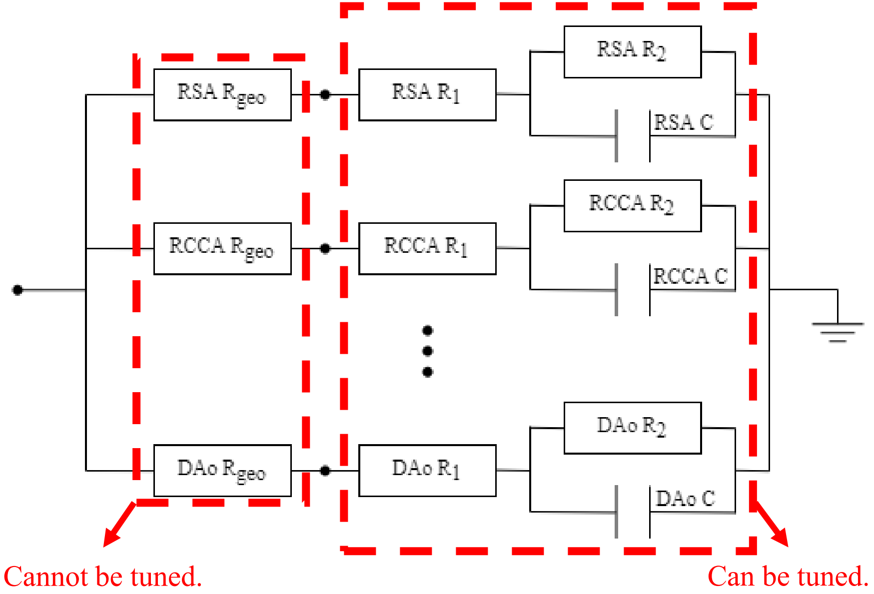
Input parameter:
Flowrate waveform: $Q(t)$

Controllable parameters:
Max, min pressure at AAo: P_{max}, P_{min};
Flow distribution of each branch: $Q\%_{RSA}$, $Q\%_{RCCA}$, $Q\%_{DAo}$, $Q\%_{LCCA}$, $Q\%_{LSA}$

$$Q\%_i = \frac{\int_0^T Q_i(t)dt}{\int_0^T Q_{AAo}(t)dt}$$

Where i suggests each branch.





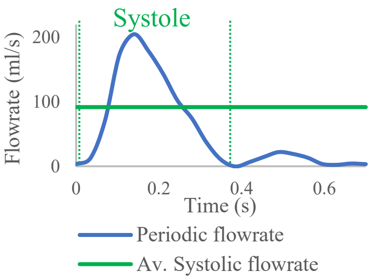
Find R_{geo} → Tune Windkessel model parameters

Flow distribution can be controlled by:

$$Q\%_{RSA} : Q\%_{RCCA} : \dots : Q\%_{DAo} = \frac{1}{R_{total_{RSA}}} : \frac{1}{R_{total_{RCCA}}} : \dots : \frac{1}{R_{total_{DAo}}}$$

For each branch: $R_{total} = R_{geo} + R_1 + R_2$

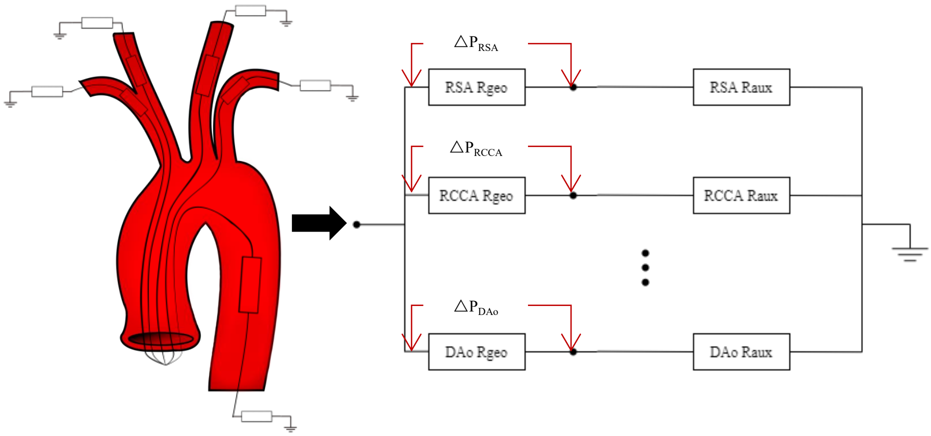
Find geometric resistance



Case	RSA R_{geo}	RCCA R_{geo}	DAo R_{geo}	LCCA R_{geo}	LSA R_{geo}
Steady	7.66E-3	3.45E-1	7.73E-2	6.96E-2	1.76E-1
Periodic	4.51E-4	3.31E-1	7.36E-2	6.67E-2	1.69E-1

Resistance unit in mmHg*s/ml

R_{geo} in both cases are similar, to simplify the algorithm, steady simulation is used.



1. Tune auxiliary resistances according to $Q\%_{RSA}:\ldots:Q\%_{DAo} = \frac{1}{R_{aux_{RSA}}}:\ldots:\frac{1}{R_{aux_{DAo}}}$.

2. Run steady flow CFD simulation with R_{aux} .

3. Calculate R_{geo} for each downstream branch.

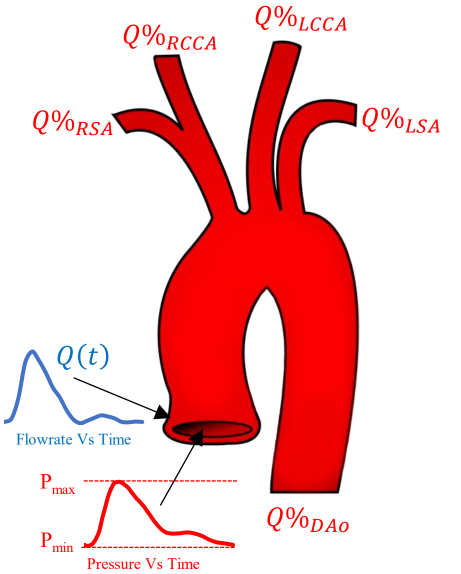
R_{aux} is to control flow distribution of each branch.

$$Q\%_{RSA}:Q\%_{RCCA}:\ldots:Q\%_{DAo} = \frac{1}{R_{total_{RSA}}}:\frac{1}{R_{total_{RCCA}}}:\ldots:\frac{1}{R_{total_{DAo}}}$$

$$R_{total} = R_{geo} + \boxed{R_{\mathrm{aux}}} \uparrow$$

Find Windkessel model parameters

Input value Controllable value



i denotes each branch.

1. Get total resistance for branch i:

$$R_{total_i} = \frac{P_{mean}}{Q_{mean_i}}$$

Where, $P_{mean} = \frac{1}{3}(P_{max} - P_{min}) + P_{min}$;

$$Q_{mean_i} = \frac{\int_0^T Q(t)dt}{T} * Q\%_i$$

2. Get Windkessel model resistance and max min pressure for branch i:

$$R_{WKM_i} = R_{total_i} - R_{geo_i}$$
$$P_{max_i} = P_{max} - Q_{mean_i} R_{geo_i}$$
$$P_{min_i} = P_{min} - Q_{mean_i} R_{geo_i}$$

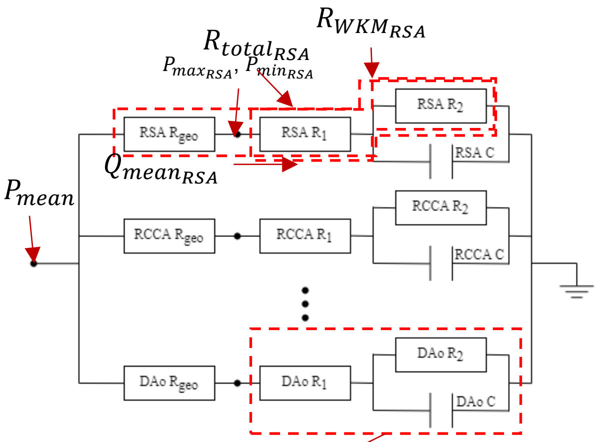
3. Find R_1, R_2, and C for branch i by the pattern search.

$R_{1_i} = 0\sim R_{WKM_i}$; $R_{2_i} = R_{WKM_i} - R_{1_i}$; $C_i = 0\sim3$ml/mmHg

Objective function:

$$obj = \sqrt{\frac{1}{2}[(P_{max_i} - P_{max_{ODE}})^2 + (P_{min_i} - P_{min_{ODE}})^2]}$$

Get a set of R_1, R_2, and C for all branches.



Possible set

$Q(t)$ | R_1, R_2, C

Windkessel model ODE

$$\left(1 + \frac{R_1}{R_2}\right) Q(t) + C R_1 \frac{\partial Q(t)}{\partial t} = \frac{P(t)}{R_2} + C\frac{\partial P(t)}{\partial t}$$

$P(t)$: $P_{max_{ODE}}$ $P_{min_{ODE}}$

Global optimization

Get parameter set with min obj

Simulation setup

Fluid settings:

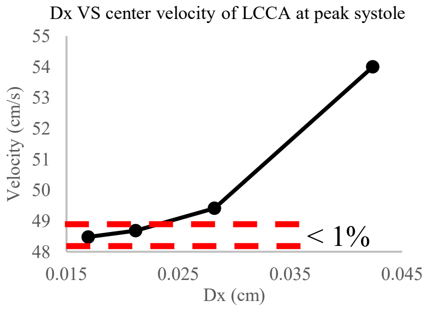
Newtonian fluid;
Density: 1060 kg/m^3;
Kinematic viscosity 3.3e-6 m^2/s.

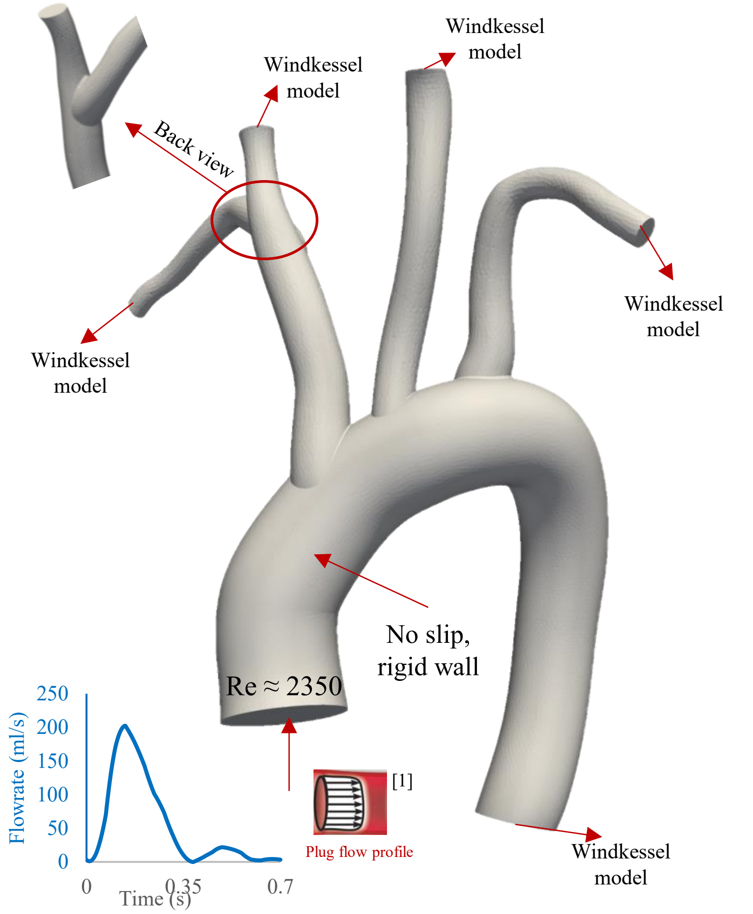
Boundary condition:

No slip and rigid wall;
Plug profile periodic velocity inlet;
Flowrate variant pressure outlet with three element Windkessel model.

Mesh Independence study:

When dx <= 0.02 cm, the difference is smaller than 1%. In the simulation, there are about 21 million fluid cells.

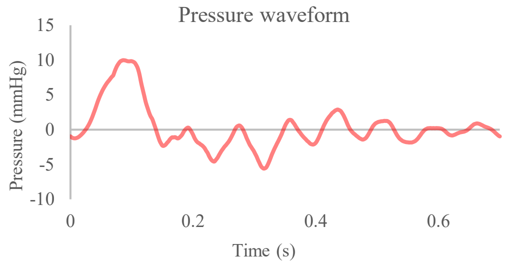


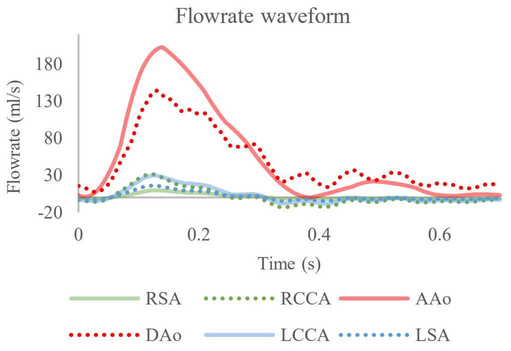


[1] http://samples.jbpub.com/9780763779351/79351_CH04_BulwerSec.pdf

Results

Constant zero pressure outlets

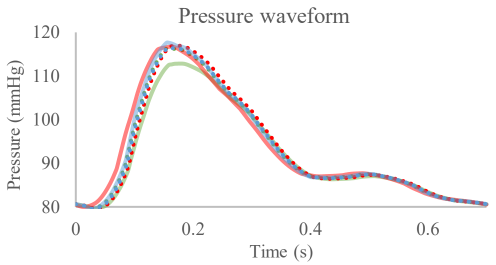


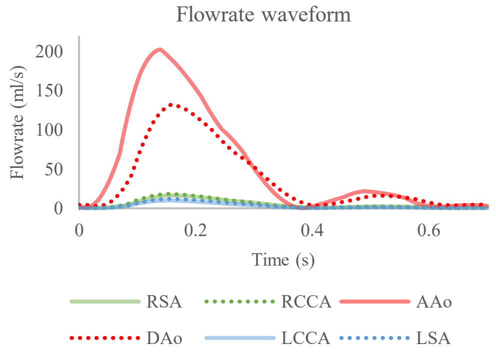


Flow distribution without Windkessel model

Branch	DAo	RSA	RCCA	LCCA	LSA
Distribution	90.06%	2.08%	0.84%	4.82%	2.48%

Our algorithm with Windkessel model





$$L2\ norm\ error = \sqrt{\frac{1}{n}\sum_{i}(val_{sim} - val_{set})^2}$$

Where val_{sim} is the value in simulation; val_{set} is the pre-set value; n is the amount of branch.

Flow distribution

Branch	Pre-set	Simulation
DAo	69.10%	68.81%
RSA	9.50%	9.36%
RCCA	9.80%	9.69%
LCCA	5.20%	5.17%
LSA	6.40%	6.33%
L2 norm error		**0.16%**

Pressure range

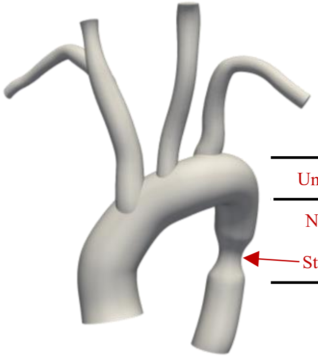
Unit: mmHg	Pre-set	Simulation
Max pressure	120	119.56
Min Pressure	80	79.60
L2 norm error		**0.42**

Windkessel model parameters

Branch	R_1	R_2	C
DAo	0.26	0.35	1.43
RSA	1.91	2.36	0.19
RCCA	1.85	2.45	0.20
LCCA	3.50	4.66	0.11
LSA	2.84	3.72	0.13

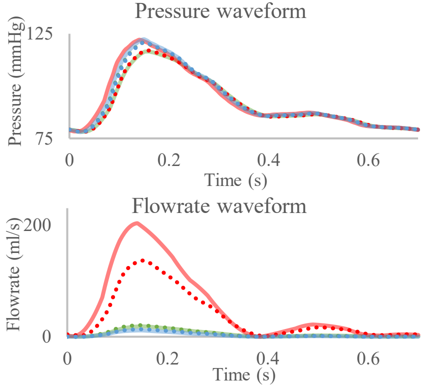
Resistance unit: mmHg*s/mL Compliance unit: mL/mmHg

Results for simulation with stenosis or even distribution



Geometry with stenosis

Unit: cm^2	Area
Normal	2.90
Stenosis	0.98



Flow distribution

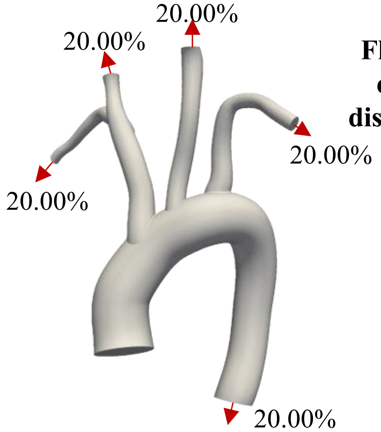
Branch	Pre-set	Simulation
DAo	69.10%	68.61%
RSA	9.50%	9.42%
RCCA	9.80%	9.75%
LCCA	5.20%	5.21%
LSA	6.40%	6.37%
L2 norm error		**0.22%**

Windkessel model parameters

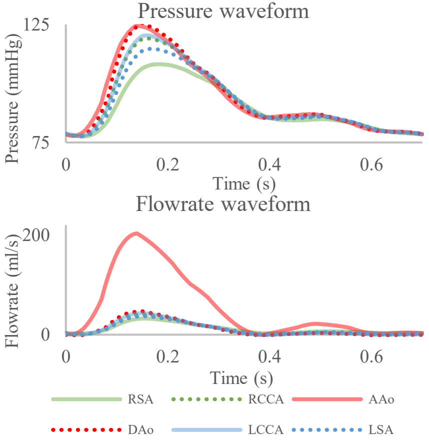
Branch	R_1	R_2	C
DAo	0.26	0.33	1.41
RSA	1.91	2.36	0.19
RCCA	1.85	2.45	0.20
LCCA	3.50	4.66	0.11
LSA	2.84	3.72	0.13
Resistance unit: mmHg*s/mL		Compliance unit: mL/mmHg	

Pressure range

Unit: mmHg	Pre-set	Simulation
Max pressure	120	122.75
Min Pressure	80	78.76
L2 norm error		**2.14**



Flowrate evenly distributed



Flow distribution

Branch	Pre-set	Simulation
DAo	20.00%	20.38%
RSA	20.00%	19.98%
RCCA	20.00%	19.97%
LCCA	20.00%	20.13%
LSA	20.00%	19.79%
L2 norm error		**0.20%**

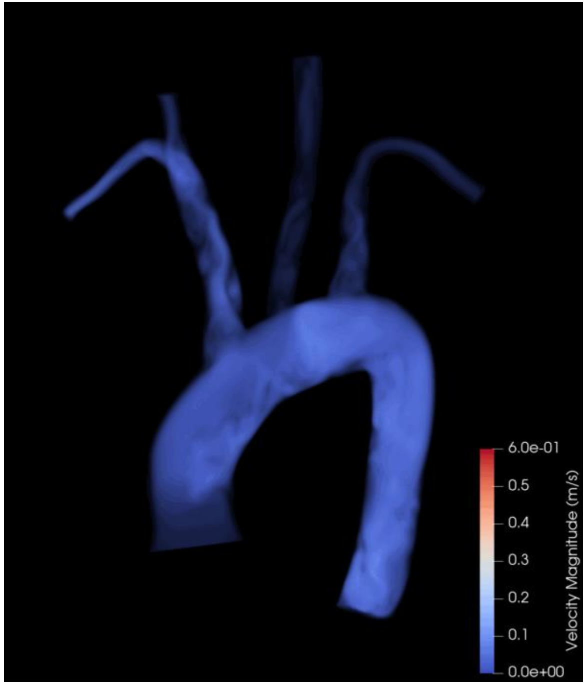
Windkessel model parameters

Branch	R_1	R_2	C
DAo	0.91	1.21	0.42
RSA	0.89	0.89	0.38
RCCA	0.91	1.15	0.41
LCCA	0.91	1.15	0.41
LSA	0.90	1.05	0.40
Resistance unit: mmHg*s/mL		Compliance unit: mL/mmHg	

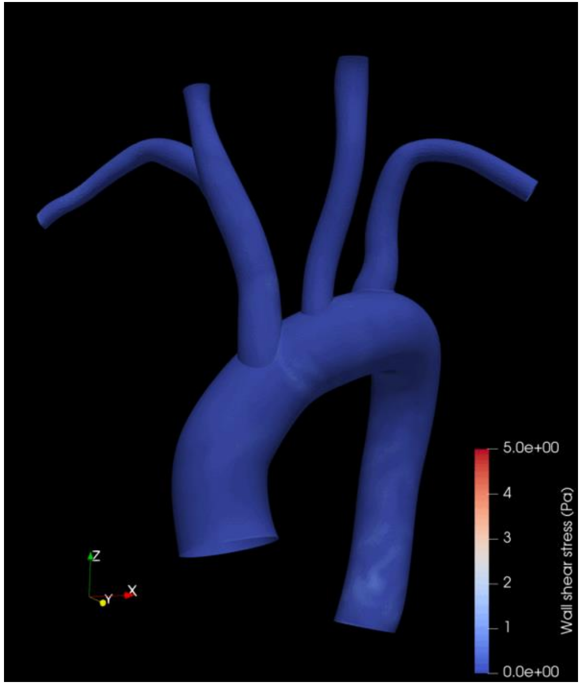
Pressure range

Unit: mmHg	Pre-set	Simulation
Max pressure	120	125.64
Min Pressure	80	78.49
L2 norm error		**4.13**

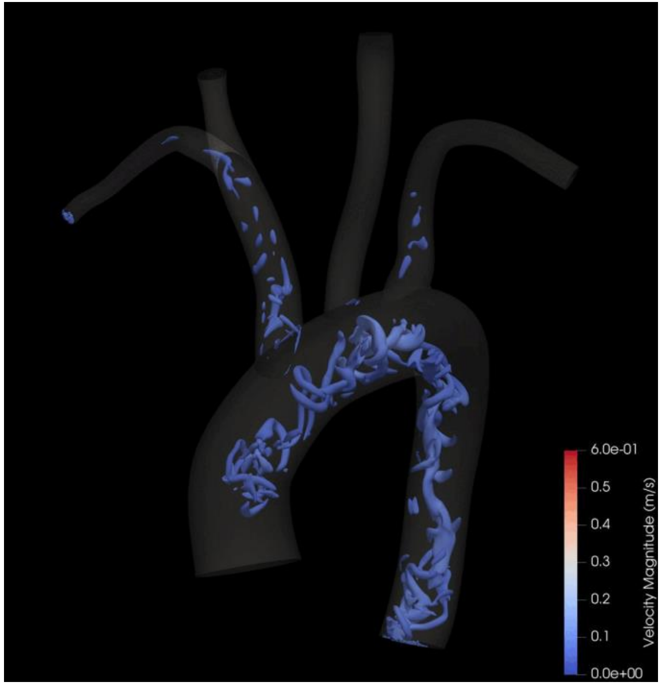
Results: animation from normal geometry with physiological flow distribution



Velocity magnitude field



Wall shear stress



Lambda 2

Conclusion:

We developed a fast algorithm to approach optimal Windkessel parameters for patient-specific aortic flow simulations:

1. Find geometric resistances by a steady CFD simulation.
2. Get the optimized Windkessel model parameters by the global optimization with the consideration of R_{geo}.
3. Run periodic flow simulation with tuned Windkessel model.

By this algorithm, max, min pressure of ascending aorta and flow distribution of each downstream branch are controllable.

Thank you!

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Appendix

Windkessel model parameters			
Branch	R_1	R_2	C
DAo	0.27	2.20	1.53
RSA	1.96	15.98	0.21
RCCA	1.90	15.50	0.22
LCCA	3.59	29.20	0.11
LSA	2.92	23.73	0.14
Resistance unit: mmHg*s/mL		Compliance unit: mL/mmHg	

For 80~120mmHg with normal geometry and normal flowrate distribution

Windkessel model parameters			
Branch	R_1	R_2	C
DAo	0.14	2.21	0.64
RSA	0.78	12.23	0.12
RCCA	0.89	13.80	0.10
LCCA	0.89	13.80	0.10
LSA	0.73	11.40	0.12
Resistance unit: mmHg*s/mL		Compliance unit: mL/mmHg	

Madhavan S, Kemmerling E M C. The effect of inlet and outlet boundary conditions in image-based CFD modeling of aortic flow[J]. Biomedical engineering online, 2018, 17(1): 1-20.