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An Immersed Boundary - Lattice Boltzmann Method for Fluid-Structure Interaction Modeling

Zongze Li
Supervisor: Dr. Mao, Wenbin

Florida Fluids Symposium III, Embry Riddle Aeronautical University, Daytona Beach, FL
May 10, 2024

Introduction

- Fluid-structure interaction (FSI) is common and understand the mechanism can unlock the innovation of bionic field.
- Lattice Boltzmann method (LBM) has gained the popularity on its simple implementation and inherently parallel running feature.
- Immersed boundary method (IBM):
  - Internal fluid effect (from the fluid enclosed by the geometry) becomes a major error source contaminating the accuracy of the solid body simulation, especially at high Reynolds number ($Re > 50$).
  - Some algorithms to eliminate this error have existed in the field, e.g., Feng's and Lagrangian points approximation algorithms.

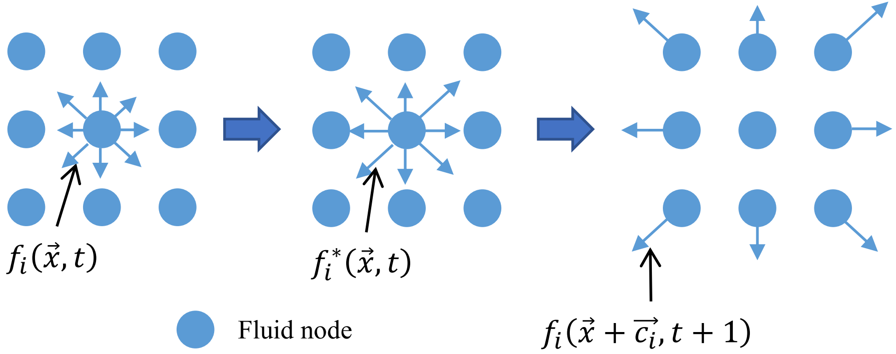
- Objective: Build up a FSI solver to simulate biological fluid mechanics, such as sea butterfly swimming.

Lattice Boltzmann Method (LBM)

Lattice Boltzmann equations:

$$f_i^*(\vec{x},t) = f_i(\vec{x},t) + \Omega_i(\vec{x},t)$$
$$f_i(\vec{x}+\vec{c_i},t+1) = f_i^*(\vec{x},t)$$

Collision: Streaming:



In LBM:

Relaxation time: $\tau = \frac{\nu}{c_s^2} + \frac{\Delta t}{2}$ Density: $\rho = \sum_i f_i$

Pressure: $p = \rho c_s^2$

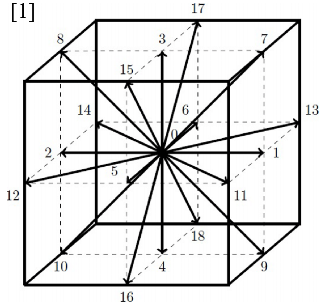
Velocity: $\rho\vec{u} = \sum_i f_i \vec{c_i}$

BGK collision operator:

$$\Omega_i(\vec{x},t) = -\frac{f_i(\vec{x},t) - f_i^{eq}(\vec{x},t)}{\tau}$$

$$f_i^{eq}(\vec{x},t) = \omega_i \rho\left[1 + \frac{\vec{u}^{eq}\cdot\vec{c_i}}{c_s^2} + \frac{(\vec{u}^{eq}\cdot\vec{c_i})^2}{2c_s^4} - \frac{\vec{u}^{eq}\cdot\vec{u}^{eq}}{2c_s^2}\right]$$

[1]



D3Q19 lattice

For external force $\vec{F}(\vec{x})$, we use Shan/Chen[2] force correction term:

$$\vec{u}(\vec{x}) = \frac{1}{\rho}\sum_i f_i \vec{c}_i + \frac{\vec{F}(\vec{x})}{2\rho}\Delta t$$

$$\vec{u}^{eq}(\vec{x}) = \frac{1}{\rho}\sum_i f_i \vec{c}_i + \frac{\tau}{\Delta t}\frac{\vec{F}(\vec{x})}{\rho}\Delta t$$

Immersed Boundary Method (IBM)

1. Velocity interpolation at Lagrangian vertex $\vec{X}$:

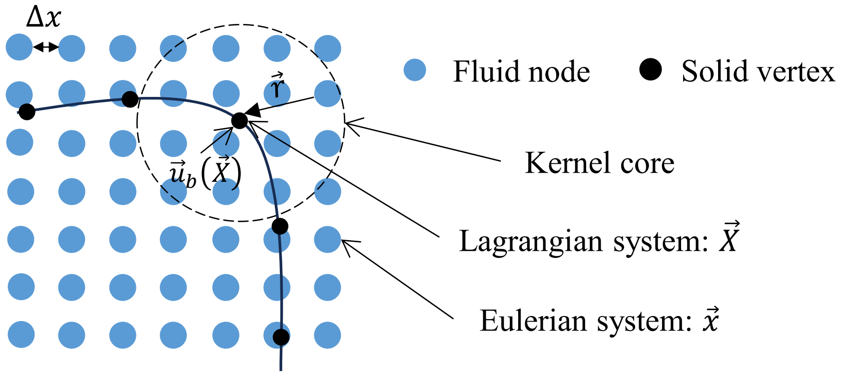
$$\vec{u}(\vec{X}) = \sum_{\vec{x}} \Delta x^3 \vec{u}(\vec{x}) \delta(\vec{X}, \vec{x})$$

2. Force calculation at Lagrangian vertex $\vec{X}$:

$$\vec{F}(\vec{X}) = 2\rho(\vec{X}) \frac{\vec{u}_b(\vec{X}) - \vec{u}(\vec{X})}{\Delta t}$$

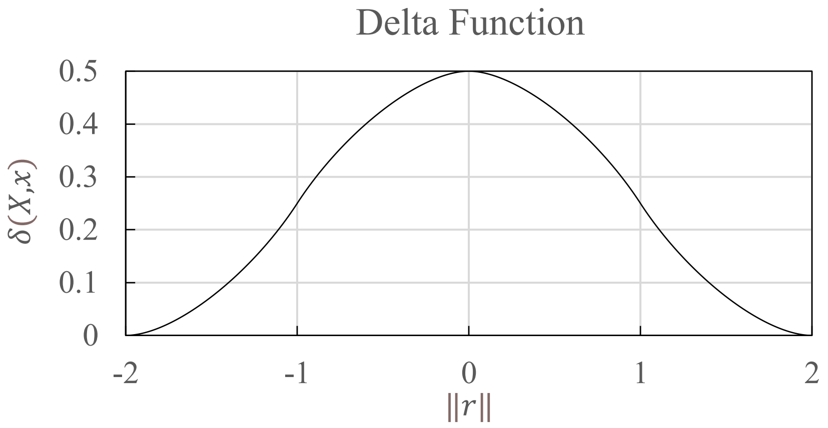
3. Force spreading to Eulerian fluid nodes in the kernel core:

$$\vec{F}(\vec{x}) = \sum_{\vec{X}} \vec{F}(\vec{X}) \delta(\vec{X}, \vec{x})$$



Delta function ($\vec{r} = \vec{X} - \vec{x}$):

$$\delta(\vec{X}, \vec{x}) = \begin{cases} \frac{1}{8}\left(3 - 2\|\vec{r}\| + \sqrt{1 + 4\|\vec{r}\| - 4\|\vec{r}\|^2}\right) & 0 \le \|\vec{r}\| \le \Delta x \\ \frac{1}{8}\left(5 - 2\|\vec{r}\| - \sqrt{-7 + 12\|\vec{r}\| - 4\|\vec{r}\|^2}\right) & \Delta x < \|\vec{r}\| < 2\Delta x \\ 0 & \|\vec{r}\| \ge 2\Delta x \end{cases}$$



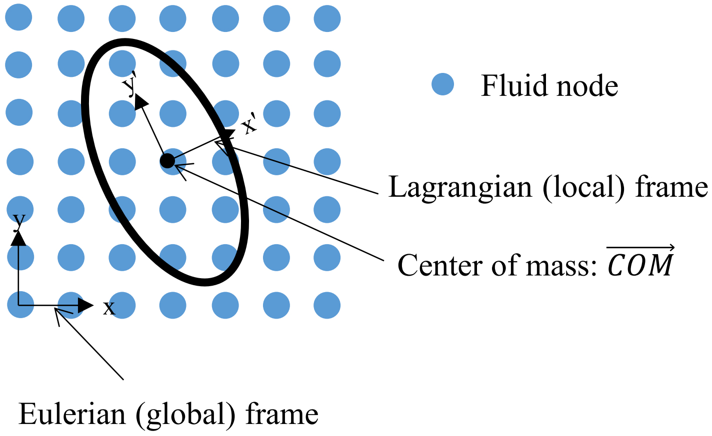
Rigid Body Motion

Governing Equations:

Translational velocity: $M\frac{d\vec{U}}{dt} = \vec{F}$

Angular velocity: $\boldsymbol{I}_B\frac{d\vec{\Omega}_B}{dt} + \vec{\Omega}_B \times (\boldsymbol{I}_B\vec{\Omega}_B) = \boldsymbol{S}^T\vec{T}$

M: mass of solid body
$\boldsymbol{I}_B$: inertia matrix of solid body
Subscript B: the property about the local frame



Force on solid: $\vec{F} = \sum_{\vec{X}} \vec{F}(\vec{X})$

Torque on solid: $\vec{T} = \sum_{\vec{X}} (\vec{X} - \overrightarrow{COM}) \times \vec{F}(\vec{X})$

Rotation matrix $\boldsymbol{S}$:

$$\boldsymbol{S} = \begin{bmatrix} q_0^2 - q_2^2 - q_3^2 + q_1^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_3^2 - q_1^2 + q_2^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

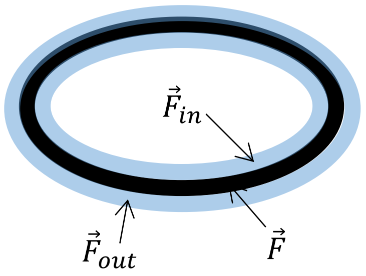
Where

Quaternion unit vector (orientation of the geometry):

$\vec{Q} = (q_0, q_1, q_2, q_3)$

Internal Fluid Effect Elimination

Force around the shell:



$$\vec{F} = \vec{F}_{in} + \vec{F}_{out}$$

$$\vec{T} = \vec{T}_{in} + \vec{T}_{out}$$

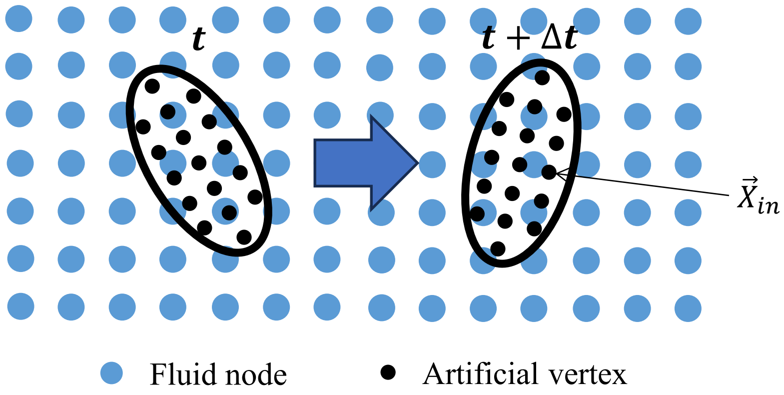
Feng's approximation[3]:

Governing equations of rigid body motion:

$$\vec{U}_B^{t+\Delta t} = \left(1 + \frac{\rho_F}{\rho_S}\right)\vec{U}_B^{t} - \frac{\rho_F}{\rho_S}\vec{U}_B^{t-\Delta t} + m_S^{-1}\boldsymbol{S}^T\vec{F}\Delta t$$

$$\vec{\Omega}_B^{t+\Delta t} = \left(1 + \frac{\rho_F}{\rho_S}\right)\vec{\Omega}_B^{t} - \frac{\rho_F}{\rho_S}\vec{\Omega}_B^{t-\Delta t} + \boldsymbol{I}_{\boldsymbol{S}_{\boldsymbol{B}}}^{-1}\left[\boldsymbol{S}^T\vec{T} + \vec{\Omega}_B^{t-\Delta t} \times \left(\boldsymbol{I}_{\boldsymbol{F}_{\boldsymbol{B}}}\vec{\Omega}_B^{t-\Delta t}\right) - \vec{\Omega}_B^{t} \times \left(\boldsymbol{I}_{\boldsymbol{S}_{\boldsymbol{B}}}\vec{\Omega}_B^{t}\right)\right]\Delta t$$

Lagrangian Points Approximation (LPA)[4]:



$$\vec{P}_{in}(t) = \sum_{\vec{X}_{in}} \vec{u}(\vec{X}_{in}, t)\Delta V_{in}$$

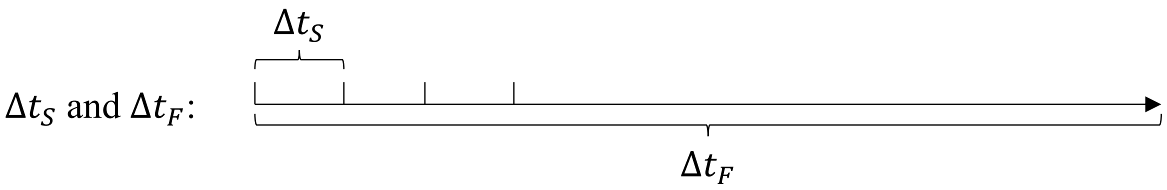
$$\vec{L}_{in}(t) = \sum_{\vec{X}_{in}} (\vec{X}_{in} - \overrightarrow{COM}) \times \vec{u}(\vec{X}_{in}, t)\Delta V_{in}$$

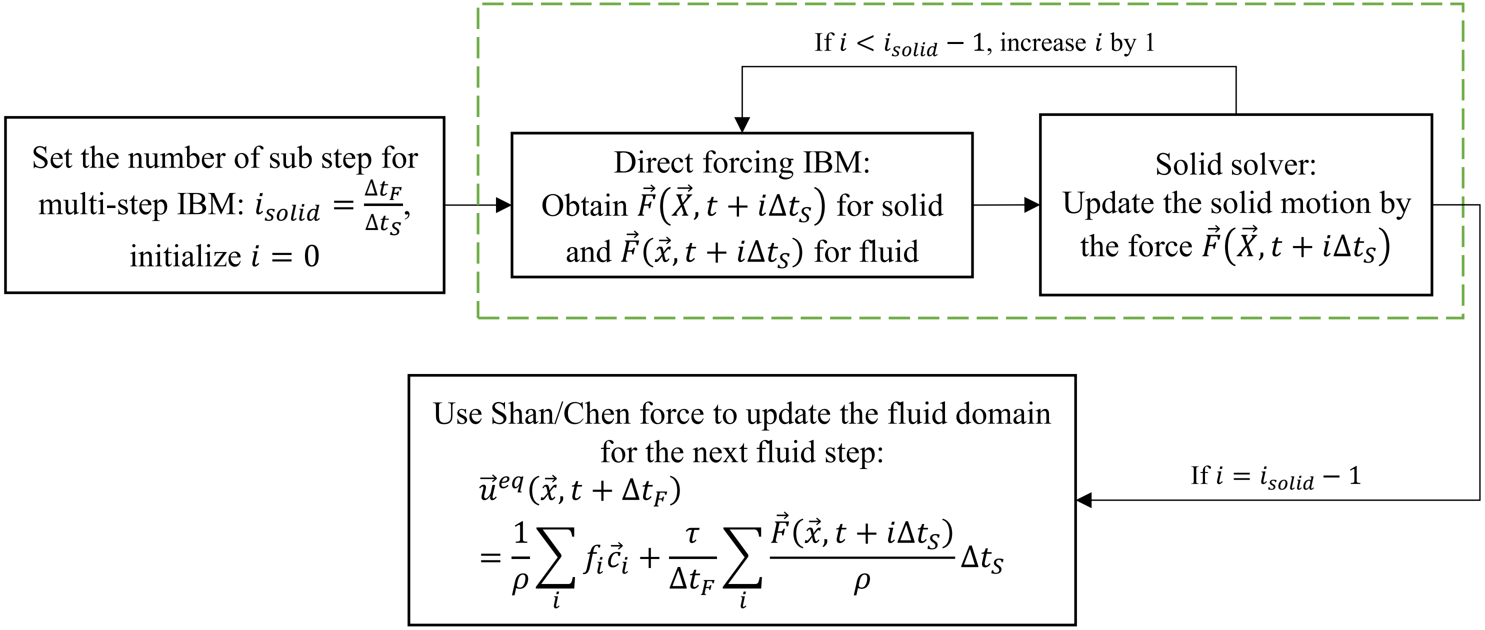
$$\vec{F}_{in} = -\frac{\vec{P}_{in}(t) - \vec{P}_{in}(t-\Delta t)}{\Delta t}$$

$$\vec{T}_{in} = -\frac{\vec{L}_{in}(t) - \vec{L}_{in}(t-\Delta t)}{\Delta t}$$

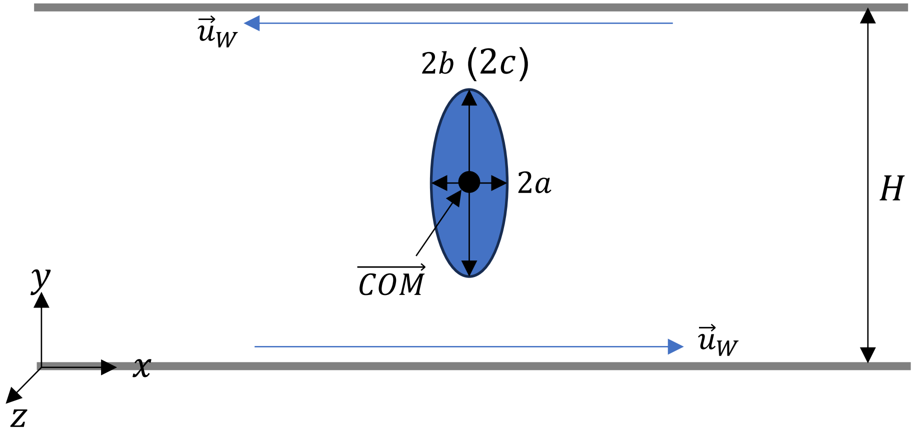
Multi-Step IBM

Mitigate instability when coupling fluid and solid solvers.





Validation: Oblate spheroid in linear shear flow[5]



Simulation Setup

Domain size: 200*80*40 LB unit

Center of mass ($\overrightarrow{COM}$) position: (100, 40, 20)

Neutrally buoyant spheroid: $\frac{\rho_S}{\rho_F} = 1$

Spheroid size:

$a = 8 \quad \mathrm{b} = 16 \quad c = 16$

Empirical solution

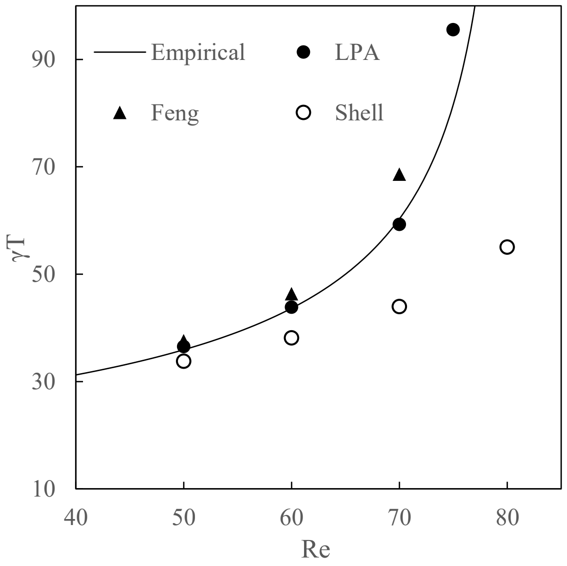
Period (T): $\gamma T = 200(81 - Re)^{-0.5}$

$$Re = \frac{4\rho b^2 \gamma}{\mu}$$

$$\gamma = \frac{2u_W}{H}$$

Results: Oblate spheroid in linear shear flow

Period vs. Re for two internal fluid effect elimination algorithms



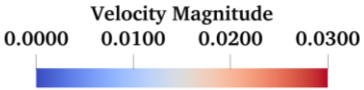
Error for each algorithm

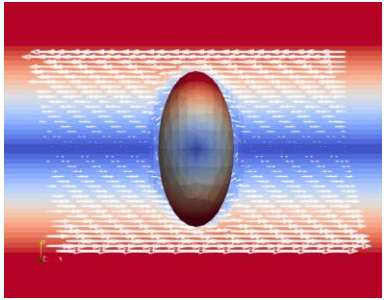
Re	LPA	Feng
50	1.66%	4.77%
60	0.49%	6.24%
70	1.66%	13.85%
75	17.05%	Inf

Stability of shell solid simulation

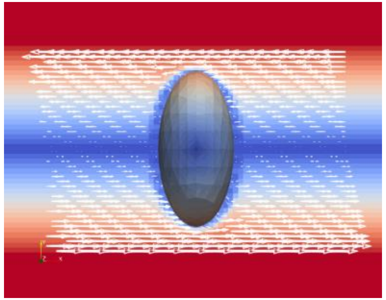
Thickness	i_{solid}	Stable?
0.1	4	No
0.1	8	No
0.1	10	Yes
0.1	16	Yes

$$Error = \frac{|numerical - empirical|}{empirical}$$

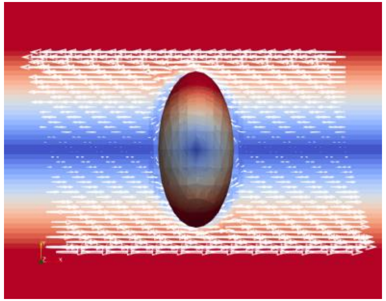




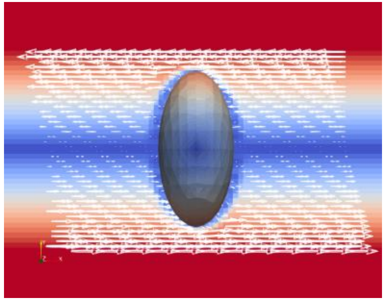
Re = 70, LPA



Re = 70, Feng

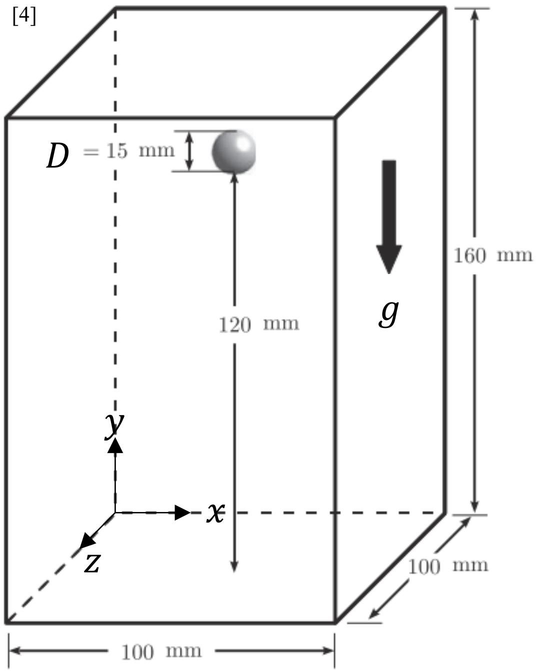


Re = 75, LPA



Re = 75, Feng

Validation: Sediment of a sphere[6]



Simulation Setup

Domain size (depth*width*height): 100*100*160 mm

Diameter of sphere (D): 15 mm

Density of sphere (ρ_S): 1120 $\frac{kg}{m^3}$

Sphere center initial position: (50 mm, 50 mm, 127.5 mm)

Gravity $g = 9.8\frac{m}{s^2}$

Test cases

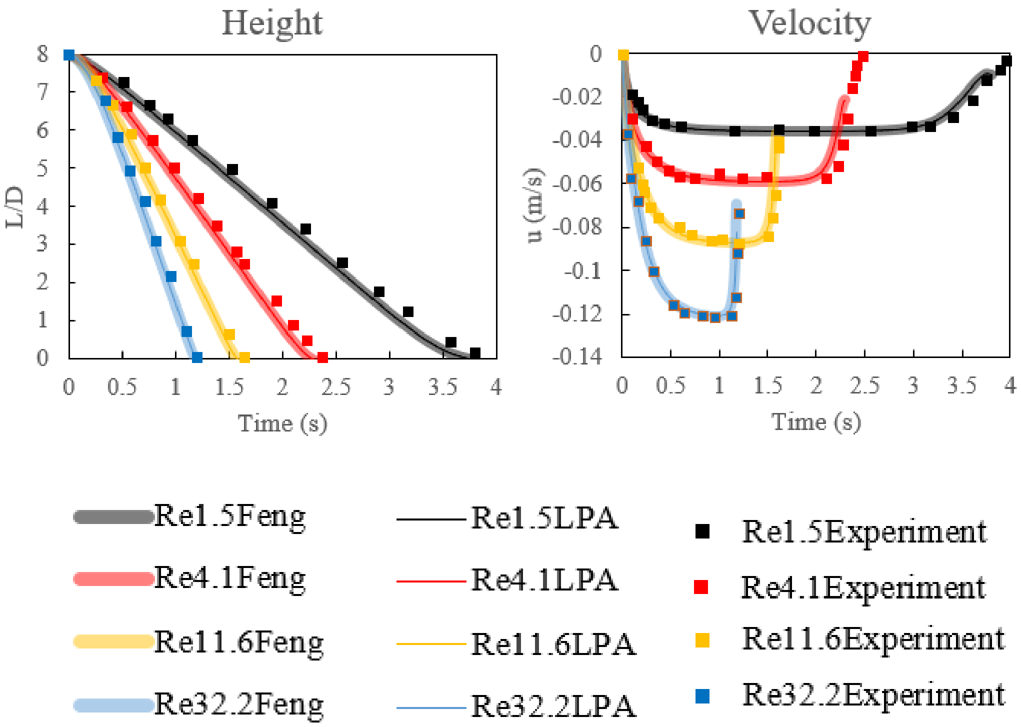
Re	ρ_f (kg/m^3)	$\mu(\times 10^{-3}$Ns/m^2)
1.5	970	373
4.1	965	212
11.6	962	113
32.2	960	58

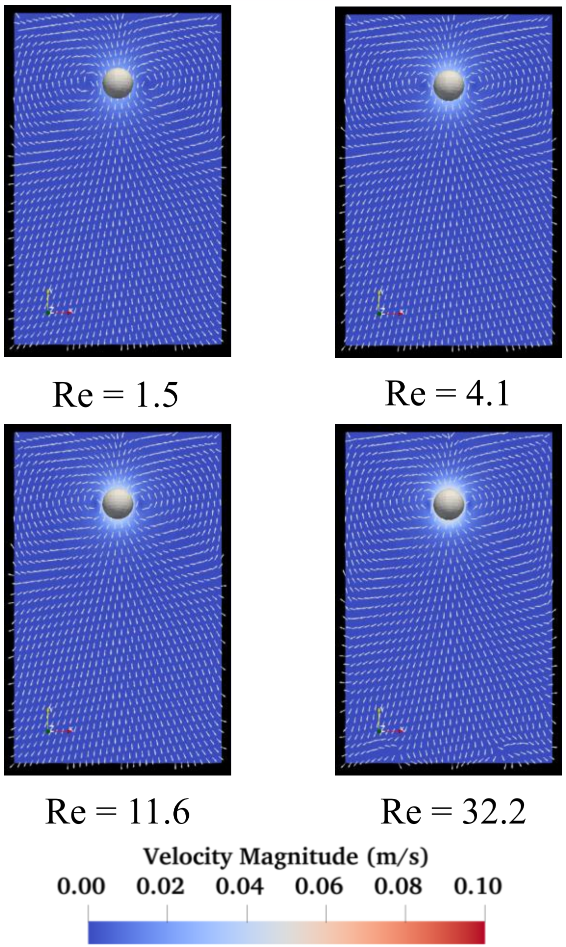
$$Re = \frac{\rho_f u_\infty D}{\mu}$$

u_∞: final velocity

Results: Sediment of a sphere

Trajectory and velocity under different Re with different internal fluid effect eliminations





Conclusion:

- Multi-step IBM can help to stabilize the FSI simulations using the thin shell geometry.
- Demonstrate internal fluid effect under higher Reynolds number ($Re > 50$).
- LPA maintains a good estimation of internal hydrodynamics for high Reynolds number ($Re \leq 70$).
- Feng's algorithm provides competitive accuracy when Reynolds number is small ($Re < 50$).

Future work:

- May further analyze and find a better way to eliminate the internal effect for larger Reynolds number.
- Apply our FSI solver to simulate sea butterfly swimming.

Reference:

[1] Ladd A J C. Numerical simulations of particulate suspensions via a discretized Boltzmann equation. Part 1. Theoretical foundation[J]. Journal of fluid mechanics, 1994, 271: 285-309.

[2] Shan, X. and H. Chen, Lattice Boltzmann model for simulating flows with multiple phases and components. Physical review E, 1993. 47(3): p. 1815.

[3] Feng Z G, Michaelides E E. Robust treatment of no-slip boundary condition and velocity updating for the lattice-Boltzmann simulation of particulate flows[J]. Computers & Fluids, 2009, 38(2): 370-381.

[4] Suzuki K, Inamuro T. Effect of internal mass in the simulation of a moving body by the immersed boundary method[J]. Computers & Fluids, 2011, 49(1): 173-187.

[5] Aidun CK, Lu Y, & Ding EJ (1998) Direct analysis of particulate suspensions with inertia using the discrete Boltzmann equation. J Fluid Mech 373:287-311.

[6] ten Cate A, Nieuwstad CH, Derksen JJ, van den Akker HEA. Particle imaging velocimetry experiments and lattice-Boltzmann simulations on a single sphere settling under gravity. Phys Fluids 2002;14:4012–25.

Thank you!

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